

The Concordance Model - a Heuristic Approach from a Stationary Universe

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The line element of a spatially Euclidean universe is given in a general scalar form $ds = \zeta^* ds_{\text{SRT}}$, as fixed by the requirement of a constant intergalactic speed of light $c^* = c$. The special assignment $\zeta^* = e^{Ht^*}$ leads to a stationary universe. Thus, the redshift parameter $z = e^{Ht^*} - 1$ is independent of time for galaxies statistically at rest. By itself, this model is near the observational facts. With the cosmological parameters $\Omega_\Lambda = 0.737$ (dark energy), $\Omega_M = 0.263$ (matter) straightforwardly fitted from this embedding universe, today's evolutionary concordance model is met perfectly.

A) Situation – As is well known, a wealth of outstanding cosmological discoveries was made in the last years [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17] (this list incomplete, for a compendium see e.g. [18], and more references therein). The surprise was excellent – as well as the consistency, too, which led to the ‘concordance (consensus) model’ (CCM) of today's cosmology.

This model is governed by a spatially flat line element of Friedman-Lemaître-Robertson-Walker (FLRW) form [19], [20], [21], [22], [23], [24], with a matter density $\rho_M \approx 0.27\rho_c$ (including dark matter), and an amount of dark energy $\Lambda/\kappa \equiv \rho_\Lambda = \rho_0 - \rho_M \approx 0.73\rho_c$ due to a cosmological constant Λ . Here $\rho_0 \equiv \rho_{\text{total}} \approx \rho_c$ with $\rho_c = 3H^2/\kappa c^2$ the critical density, κ the gravitational, and $H (\equiv H_0)$ Hubble's *constant* [4], [25] today [in contrast to Hubble's *parameter* $H_p(t)$, s. below]. The present ‘deceleration’ parameter is q_0 , and T_0 the age of the ‘universe’.

Though the underlying big-bang concept has proved to be exceptionally fruitful indeed – first of all by its prediction of a cosmic microwave background (CMB) [26] – there remain several well known questions concerning the singularity, inflation, horizons, coincidences, and fine tuning, for example. In contrast to most corresponding attempts, however, this paper is focused on what (general) relativity theory, (G)RT, might be able to tell *without* any additional hypotheses about the universe. Thereby, of course, one should keep in mind that the energy-stress tensor of Einstein's gravitational equations deals with purely phenomenological densities only (what allows for the equivalence principle), thus implying no information about its composition at all.

As many physicists may feel, there should be a relativistic concept of a stationary universe – though embedding an evolutionary cosmos if necessary. Unfortunately such an attempt seems blocked since the Steady-state Theory [27], [28] has turned out to conflict with important observational facts [29]. In spite of its intention, however, this theory is not stationary at all. In particular its redshift parameter z and thereby the cosmic distances (together with all observables depending on z) would be functions of time. – Nevertheless, in the following we shall see that there is a truly stationary line element (1) of GRT, which by itself is near the observational facts (2) – (5) and, after transformed to its FLRW-form (6) – (8), allows to fit in the CCM cosmos (9), (10) straightforwardly.

B) Preliminaries – The universal (coordinate) time t^* is understood to be that scale on which far away from local inhomogeneities the universal (coordinate) speed of light is constant $c^* = c$. The universal (Euclidean) space is assumed to be filled with a homogeneous and isotropic ultra-large scale distribution of matter and energy. Its coordinate system is $l^{*\alpha}$ ($\alpha, \beta, \dots = 1, 2, 3$ in contrast to $i, k, \dots = 0 \dots 3$). – Since the cosmic evolution seems to affect ‘our’ cosmos from a joint beginning, it may be appropriate to distinguish cosmos from universe (stationary the last, though on scales large enough only).

Timescales as used in the following are: the *universal* (coordinate) time t^* , $T^* \equiv t^* - T_H$ where $T_H = 1/H$ (not to be confused with an old scale of atomic time), the *adapted* coordinate time t' , $T' \equiv T_H + t'$, then the *integrated* coordinate time t , $T \equiv T_H + t$ (not to be confused with T_i^i the trace of the energy-stress tensor T_i^k , or the absolute temperature Θ), and finally the *local* (proper) time $\tau \ll T_H$. – In particular t is the simplest, t' is the best quasi-Minkowskian coordinate time approximating local proper time τ in bounded space-time regions.

C) A stationary universe – The definition of universal time t^* above tacitly takes for granted a spatially Euclidean universe, including that every *universal* line element can be written in a simple scalar form

$$ds^{*2} = \zeta^{*2} \left\{ c^2 dt^{*2} - dl^{*2} \right\}, \quad (1a)$$

where $\zeta^* \approx 1 + Ht^* + O^2(Ht^*)$. This general scalar form is fixed uniquely by the *postulate (I) of a constant intergalactic speed of light* $c^* = c$ [which is not given in any form other than (1a)]. Now, with the special assignment

$$\zeta^* = e^{Ht^*}, \quad (1b)$$

valid from here, the general scalar form (1a) together with its energy-stress tensor T_{ik}^* not only turns out to be non-singular, but to fulfill a *postulate (II) of stationarity* as well:

Because of the exponential form of the time scalar e^{Ht^*} , all the resulting relative temporal changes depend solely on *differences* $\Delta t^* = t^* - t_R^*$. It is exactly this that allows to set any reference point of time to be $t_R^* = 0$ for arbitrary complexes of observation governed by (1). Adapting appropriate units respectively (once in each epoch), no special point (but a local direction) of the universal time scale is preferred.

According to (1) the local proper time element $d\tau$, measured with atomic clocks (at rest with respect to the CMB), and the local proper length element $d\lambda$, measured with spectral rods are

$$\begin{aligned} d\tau &\approx e^{Ht^*} dt^*, \\ d\lambda &\approx e^{Ht^*} dl^*, \end{aligned} \quad (2)$$

where $dt^*/d\tau = e^{Ht^*} = u^0$ is included [this time component of the stationary 4-velocity $u^i = (u^0, 0, 0, 0)$ is easily verified by solving the relativistic equations of motion $\delta[ds] = 0$ with respect to (1)].

According to the basics of relativistic cosmology, now a completely time-independent redshift parameter $z \equiv \lambda_{\text{observed}}/\lambda_{\text{emitted}} - 1$ (for starlight emitted from sources at rest) is immediately concluded from (1) and (2) to yield

$$l^* = \frac{c}{H} \ln(1+z) \Leftrightarrow z = e^{Hl^*/c} - 1 \quad (3)$$

with $l^* (= c\Delta t^*)$ the covered distance (Δt^* the light time). – With respect to (2), the simple explanation for this redshift is a difference between proper (λ, τ) and universal (l^*, t^*) length and time.

From the line element (1) the energy-stress tensor is given by Einstein's (original) equations $E_{ik} \equiv R_{ik} - \frac{1}{2}Rg_{ik} = \kappa T_{ik}$ and may here be written in the form

$$E_{ik}^* = \begin{pmatrix} \frac{2H^2}{c^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - p^* g_{ik}^* = \kappa T_{ik}^*, \quad (4)$$

demanding a negative *gravitational* pressure $p^* = -\frac{1}{3}\rho^*$, where $\rho^* \equiv T_0^{0*} = \rho_c e^{-2Ht^*}$ is the phenomenological energy density. Obviously p^* corresponds to something like a stationary changing cosmological ‘constant’. To state it explicitly, this gravitational pressure p^* *must* be negative because the walls of a large-scale box including a plenty of galaxies statistically at rest, would have to pull outwards, if those inside should not mass together after those outside had been removed. – With respect to (4) the phenomenological mass density might be only $\mu^* = \frac{2}{3}\rho^*/c^2$ (in contrast to the full energy density ρ^*).

From (3) one finds a relation for the apparent luminosity $I(z)$ of type Ia supernovae (SNe), which by itself is already near the magnitude versus redshift observations reported in [1], [2], lately e.g. in [12], [14], [16]:

$$I(z) = \frac{LH^2}{4\pi c^2} \left[(1+z) \ln(1+z) \right]^{-2}, \quad (5)$$

with L the absolute luminosity of a source. Again, this relation is independent of time (neglecting ‘local’ cosmic evolution), and not yet corrected for effects of ‘replenishing’ grey dust [30], large gravitational potentials, etc.

In spite of an infinite number of galaxies, another result from (3) is a finite average baryonic radiation density equivalent in its integral to that of a black body radiation of temperature $\Theta_{\text{effective}} = 5.4 \text{ K} \times (\frac{5}{4} \Omega_{\text{baryonic}})^{1/4}$ (roughly assessed using the radiation power per mass of the sun), thereby solving Olbers' paradox [31], too. Strange enough, with a fraction $\Omega_{\text{baryonic}} \approx 5\%$ of ρ_c this corresponds to 2.7 K for example (s. ref. of appendix).

Though it would be possible to calculate the CCM parameters below from the stationary line element in its transparent scalar form directly, it is more convenient to write (1) in a traditional FLRW-form now. This is done by simply transforming the universal time t^* to the *integrated* coordinate time t (without thereby changing any physical results). We start defining appropriate coordinates by

$$\begin{aligned} t^* &= \frac{\ln(HT)}{H} - \frac{n}{2} \frac{Hr^2}{c^2}, \\ r^* &= \frac{r}{HT}, \end{aligned} \quad (6)$$

which as *adapted* coordinates – by setting $n := 1$ and $(T, r) := (T', r')$ – transform the stationary line element (1) approximately into that of special relativity theory (SRT), except for deviations of second order $O^2(Ht', Hr'/c)$,

$$ds'^2 = e^{-(Hr'/c)^2} \left\{ \left[1 - \left(\frac{r'}{cT'} \right)^2 \right] c^2 dt'^2 + \frac{2r'(1-H^2T'^2)}{T'} dt' dr' - \left[1 - \left(\frac{H^2 r' T'}{c} \right)^2 \right] dr'^2 - d\Sigma^2 \right\}, \quad (7)$$

where $d\Sigma$ the element of a spherical surface. Then, of the *integrated* coordinates – defined by $n := 0$ in (6) – we use only the first relation to transform (1) into

$$ds_{(\text{FLRW})}^2 = c^2 dt^2 - (HT)^2 dl^{*2}, \quad (8)$$

this FLRW-form with its scale factor $a(t) \equiv 1+Ht \equiv HT$ being no longer non-singular at all. – Why so circumstantially from (1) to (8)? Because it is important to see from (7) that the adapted time t' as the *best* quasi-Minkowskian coordinate approximation to a local proper time integral τ is neither suitable to hold at and beyond cosmic distances $r' \rightarrow c/H$ nor for the time $T' \rightarrow 0$.

Indeed, both alternative representations (7) and (8) of the *stationary* universal gravitational potential yield the same results as the original form (1). It is easily verified that, in particular, the Hubble relation (3) holds from (8) in its time-independent form, too! – The stationary ‘deceleration’ parameter is $q \equiv 0$ {therefore the broken ‘ $q=0$ ’-parting-line in Fig. 4 of [32] may be used to compare relation (5) above with recent observations directly}.

Nevertheless, (1) vs. (8) suggests a contradiction. One might wonder how both should describe a stationary universe. Because of (8) it seems that at the negative Hubble time $t = -T_H = -1/H$ all proper lengths λ had been zero, all proper densities infinite, and the whole universe therefore singular. This conclusion does not seem legitimate, however, since the relation $d\lambda \approx (HT)dl^*$ holds only on scales that are local with respect to space *and* time. In addition, comparing (2) and (6) one has $d\tau \approx dt$. But the interval dt (of coordinate time) is integrable, $d\tau$ is not (since $d\tau$ is only defined locally *together* with $d\lambda$ by the relation $ds_{\text{SRT}}^2 = c^2 d\tau^2 - d\lambda^2$). Thus, even using a FLRW-form, it remains indispensable to distinguish local proper time $\tau \ll T_H$ from the FLRW-*coordinate* time t .

Given a stationary ultra-large scale universe, however, what would (2), (6) mean? The answer is, that in this view all local structures (lengths λ) are shrinking, while cosmic distances l^* – as directly measured by stationary values of z – simply stay unchanged. Nevertheless, looking *back* from our time all questions are the same as in

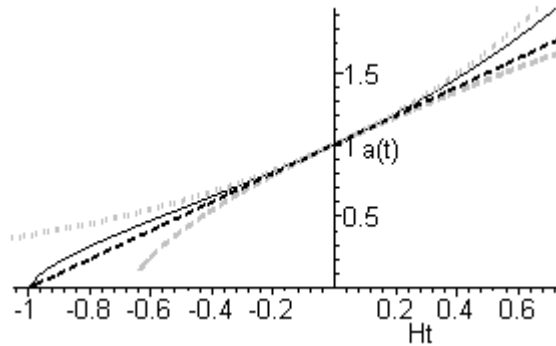
standard cosmology, too. In particular: When in the past have structures been large enough to interfere or even contact one another? – Thinking here about galaxies, clusters or superclusters for example, one would find roughly that those cannot have been permanent structures since a time earlier than about $T_1 \approx T_H/100 - T_H/10$ if they had arisen together (this period might correspond to an assumed phase of ‘re-ionization’).

To go to an extreme one may ask at what *universal* time T_α^* a structure of the modified Planck length $r_\alpha \equiv (h/ec)\sqrt{G}$ would have been as large as the Hubble-length $R_H \equiv cT_H$. The result, calculated using the integrated coordinates, would be exactly at $HT_\alpha^* \equiv \ln(r_\alpha/R_H) - 1 \equiv -1/\alpha$ (with α the fine structure constant) if for example the Hubble constant proved to be 64,7 km/s/Mpc (similar combinations yield same order results).

Since the concept of proper time and length is seen from (7) to break down at $t \rightarrow -T_H$ or $r \rightarrow cT_H$, these values might be interpreted as upper limits for macroscopic joined structures, corresponding to the Planck length and time (which represent lower ones). In a stationary universe, the singularity of the FLRW-form (8) would mean there cannot exist clocks, stars, galaxies, being older than $\tau_\alpha \approx -t_\alpha \approx 1/H$ with respect to their proper time. This cosmological concept, suggested by (G)RT, is particularly attractive because: *within physics, neither a beginning of the universe as a whole nor any eternal macroscopic joined structures seem to be plausible options.*

D) A heuristic approach to the CCM – The stationary ultra-large scale universe described by (1), neglecting all inhomogeneities and local evolution as well, should not be overinterpreted looking around in ‘our’ cosmos. – So, it might be appropriate to try and outline a possibly more realistic picture.

If it were not for the homogeneous negative gravitational pressure p^* in (4), which is completely ‘dark’ since not detectable using known physics so far (as well as any homogenous background energy density, too), one could be satisfied using (1) or (8) or even (7) to calculate observational facts. Now, what astronomers really see, is an inhomogeneous space seeded with galaxies, apparently showing a *phenomenological* pressure $p_M \approx 0$.



Given spatial flatness, assuming isotropy and homogeneity, *all* cosmic conclusions due to (G)RT may be drawn from the FLRW scale factor $a(t)$. – Top-down $(\rho_M, p_M, \rho_\Lambda) = (0, 0, 1), (0.27, 0, 0.73), (1, -1/3, 0), (1, 0, 0)$, i.e.:

Steady-state Theory $a(t) = e^{Ht}$ (prominent in the 1950s), today’s concordance model $a(t) = a_{\text{CCM}}$ [solid line, see (9)], stationary universe $a(t) = HT = 1 + Ht$ [straight broken line, see (8)],

and the Einstein-de-Sitter metric $a(t) = (1 + 3/2 Ht)^{2/3}$ (favored before the observational breakthrough last years).

It is obvious to see how near the CCM (only the range $-1 < Ht \leq 0$ is observed) is to a stationary universe, thereby fulfilling its boundary conditions at $Ht = -1, 0$ perfectly.

Since Einstein has found it an option to add an *ad-hoc* cosmological term Λg_{ik} to the right hand side of his equations, there is the chance to account for the unexplored negative gravitational pressure p^* *heuristically* (it might affect the cosmic observations and therefore, it seems, we might not be allowed to ignore it). This is done here by setting the phenomenological pressure $p_M = 0$ and using $\rho_0 \equiv \rho_c \equiv \rho_M + \rho_\Lambda$ for a spatially Euclidean FLRW-form in Einstein’s extended equations (with Λ representing the effects of the unexplored pressure p^*), what yields the scale factor for the (reasonably constrained) concordance model:

$$a_{\text{CCM}}(t) = \left\{ \left(\frac{1}{\Omega_\Lambda} - 1 \right) \sinh^2 \left[\frac{1}{2} \ln \left(\frac{1 - \sqrt{\Omega_\Lambda}}{1 + \sqrt{\Omega_\Lambda}} \right) - \frac{3}{2} \sqrt{\Omega_\Lambda} Ht \right] \right\}^{1/3}, \quad (9)$$

where $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$. This relation (9) is nothing but the scale factor for spatially Euclidean FLRW-forms, where those theoretical magnitude versus redshift relations obeying $\Omega_M + \Omega_\Lambda = 1$ ($p_M = 0$) may readily be calculated from, which (among others with $\Omega_M + \Omega_\Lambda \neq 1$) are reported in [33], [34], extended most elegantly in [35], and have been compared with observations (s. references above).

Now, in view of the embedding universe, it is natural to claim the singularity of $a_{\text{CCM}}(t)$ to be the *same* as in the FLRW-form of the stationary solution (8), i.e. $a_{\text{CCM}}(t_0 = -T_H) \stackrel{!}{=} 0$ (see figure). Because this condition is equivalent to $a_{\text{CCM}}(t_0^* = -\infty) = 0$, and again: *with respect to universal coordinates, there is no singularity at all.* – Then, from this decisive claim, which simply corresponds to $T_0 \stackrel{!}{=} 1/H_{(0)}$, the numerical solution using (9) is

$$\Omega_\Lambda = 0.737, \quad \Omega_M = 0.263, \quad (10)$$

thus almost perfectly matching the recently reported highly consistent CCM results ($\Omega_\Lambda = 0.73 \pm 0.04$ in [9], 0.72 ± 0.05 in [12], 0.75 ± 0.06 in [14]).

E) Conclusion – The simplest of all thinkable FLRW-forms (8) without cosmological constant is shown to be equivalent to the one and only *stationary* line element of GRT implying a *constant* universal speed of light (1).

According to (10), today's CCM (9) quite obviously fulfills the ‘boundary’ conditions of this stationary line element at $Ht = -1, 0$ (see figure). In particular, the CCM parameters above seem to represent nothing but the ‘normalized’ values $\rho_{\text{total}}/\rho_c \equiv 1$ (instead of 1.02, see e.g. [9]) and $H_{(0)}T_0 \equiv 1$ (instead of 0.995, *ibid.*) simply given by (8). Their heuristic deduction does not prove the CCM to be true. But it actually proves our cosmos (as described by the CCM numerically) to fit in a stationary ultra-large scale universe (as described by pure GRT).

F) Discussion – How could the clear stationarity of the simple line element (1) happen to escape its discovery even in those times, when the Steady-state Theory was discussed widely? Two important reasons may be: a) a negative *gravitational* pressure $p_M = -\frac{1}{3}\rho_M < 0$ (s. above) has not been considered a physical option before, and b) the time-independent redshift (3) with its true Hubble *constant* $H (\equiv H_0, \text{ according to the scale factor } a = HT, \text{ too})$ might have been concealed by the rather misleading Hubble parameter $H_p(T) \equiv (da/dt)/a = 1/T$ {though the stationary ‘deceleration’ is $q \equiv -a(d^2a/dt^2)/(da/dt)^2 = 0$, which value has been interpreted according to a ‘coasting’ cosmology [36], without the postulate of spatial flatness due to a constant universal speed of light $c^* = c$ }.

One may easily verify that the scale factor $a_{\text{CCM}}(t)$ above is much the same as the conventional line element $a(t) \approx (1+Ht)(1+XH^2t^2+\dots)$ e.g. with $X \approx \frac{1}{2}$ (0.5...0.75), $\Omega_\Lambda \equiv 0$, built from (8) by some complication, too. The CCM, however, raises the question: why is the matter density ρ_M of the same order as the ‘dark energy’ ρ_Λ just in our time (as emphasized in [37])? There would always remain *universal* coincidental aspects – if not finally the stationary line element (1), (8) does prove to hold by itself throughout local *cosmic* evolution everywhere.

In contrast to any inductive approach, the deductive concept of a stationary universe is supported by (G)RT without any additional hypotheses. This might help to clarify the history and the shape of ‘our’ cosmos (not necessarily isotropic everywhere), by distinguishing from the ultra-large scale background as described by (1).

Thus, in addition to all excellent agreement in local gravitational fields, relativity theory seems to be an appropriate tool also to describe a stationary universe, embedding our cosmos therein. In this view, though, one is left with the basic question, whether cosmic evolution affects what we see as a whole (according to the CCM) or ‘locally’ only (in quasars, galaxies, clusters, bubbles, or even more ‘cosmoses’, for example).

An answer – with full compatibility guaranteed – would be simply to adopt today's cosmology, though embedded in a stationary universe. Those features which at present are ascribed to a hot ‘big bang’, however, might be explained by (or applied to) *local* processes one day. In particular, any ‘big-bang’ events (possibly in connection with chaotic inflation scenarios [17], too) would take place everywhere, again and again.

Even in a stationary universe, with respect to shrinking rods and proper units one would find empty space ‘expanding’ relatively. Therefore, here would be a struggle of ultra-large scale entropic balance against gravitational creation (with local black-hole areas of decreasing entropy if necessary, temporarily delimited from external physical description) – as well as, once established, there would be a struggle of all structures against decline and decay (which might be the reason for spontaneous emission or radioactive decay processes, too).

Given a cosmic evolution as a whole, however, as indicated by the apparent change of the local CMB-temperature [38] e.g., this should not have begun earlier than from the modified Planck time $T \approx T_\alpha = T_H e^{-1/\alpha+1}$, which process might correspond to a *cosmic* inflation (started from an initial ‘fluctuation’ [11] and grown to Hubble length today), if not from a post-inflation scenario only – though in a stationary universe after all.

Appendix – A more detailed treatment of the universal scalar form (1) containing the essential results of sections B), C) [together with all numbered relations except for (9), (10)] was given in a previous e-print [39], showing that a stationary universe might be compatible with important observational facts.

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- [1] Perlmutter S., *et al.*, Bull.Am.Astron.Soc. **29** (1997) 1351, astro-ph/9812473;
Astrophys.J. **517** (1999) 565-586, astro-ph/9812133
 - [2] Riess A. G., *et al.*, Astron.J. **116** (1998) 1009-1038, astro-ph/9805201
 - [3] Carroll S. M., Liv.Rev.Rel. **4** (2001) 1, astro-ph/0004075
 - [4] Freedman W. L., *et al.*, Astrophys.J. **553** (2001) 47-72, astro-ph/0012376
 - [5] Netterfield C. B., *et al.*, Astrophys.J. **571** (2002) 604-614, astro-ph/0104460
 - [6] Turner M. S., e-print astro-ph/0108103 (2001) 1-12
 - [7] Efsthathiou G., *et al.*, MNRAS **330** (2002) 29-35, astro-ph/0109152
 - [8] Peebles P. J. E., Bahrat R., Rev.Mod.Phys. **75** (2003) 559-606, astro-ph/0207347
 - [9] Bennett C. L., *et al.*, Astrophys.J.Suppl. **148** (2003) 1, astro-ph/0302207
 - [10] Spergel D. N., *et al.*, Astrophys.J.Suppl. **148** (2003) 175, astro-ph/0302209
 - [11] Mukhanov V. F., Chibisov G. V., re-print astro-ph/0303077 (1981) 1-8
 - [12] Tonry J. L., *et al.*, Astrophys.J. **594** (2003) 1-24, astro-ph/0305008
 - [13] Guth A. H., e-print astro-ph/0306275 (2003) 1-23
 - [14] Knop R. A., *et al.*, e-print astro-ph/0309368 (2003) 1-52
 - [15] Tegmark M., *et al.*, e-print astro-ph/0310723 (2003) 1-27
 - [16] Barris B. J., *et al.*, e-print astro-ph/0310843 (2003) 1-67
 - [17] Linde A., e-print hep-th/0402051 (2004) 1-24
 - [18] Börner G., *The Early Universe – Facts and Fiction*, 4. Ed., Berlin - Heidelberg - New York 2003
 - [19] Einstein A., Sitz.ber.Preuß.Akad.Wiss. (1917) 142-152
 - [20] de Sitter W., Proc.Kkl.Akad.Amst. **19** (1917) 1217-1225; *ibid.* **20** (1917) 229-243, 1309-1312;
M.Not.Roy.Astr.Soc. **78** (1917) 3-28
 - [21] Friedman(n) A., Zeitschr.f.Physik **10** (1922) 377-386; *ibid.* **21** (1924) 326-332
 - [22] Lemaître G., Ann.Soc.Sci.Bruxelles **47** (1927) 49-59, *translated in:* M.Not.Roy.Astron.Soc. **91** (1931) 483-489;
ibid. 490-501, 703
 - [23] Robertson H. P., Astrophys.J. **82** (1935) 284-301; *ibid.* **83** (1936) 187-201, 257
 - [24] Walker A. G., Proc.Lond.Math.Soc. **42** (1936) 90-127
 - [25] Hubble E. P., Proc.N.Acad.Sci. **15** (1929) 168-173
 - [26] Alpher R. A., Bethe H., Gamow G., Phys. Rev. **73** (1948) 803-804
 - [27] Bondi W. H., Gold T., M.Not.Roy.Astron.Soc. **108** (1948) 252-270
 - [28] Hoyle F., M.Not.Roy.Astron.Soc. **108** (1948) 372-382 ; *ibid.* **109** (1949) 365-371
 - [29] Weinberg S., *Gravitation and Cosmology*, New York 1972
 - [30] Riess A. G., *et al.*, e-print astro-ph/0402512 (2004) 1-72
 - [31] Olbers H. W. M., Astron.Jahrb.1826 **51** (1823) 110-121
 - [32] Freedman W. L., Turner M. S., e-print astro-ph/0308418 (2003) 1-39
 - [33] Carroll S. M., Press W. H., Turner E. L., ARA&A **30** (1992) 499
 - [34] Goobar A., Perlmutter S., Astrophys.J. **450** (1995) 14, astro-ph/9505022
 - [35] Perlmutter S., Schmidt B. P., astro-ph/0303428 (2003) 1-24
 - [36] Kolb E. W., Astrophys.J. **344** (1989) 543-550
 - [37] Carroll S. M., e-print astro-ph/0310342 (2003) 1-22
 - [38] Srianand R., Petitjean P., Ledoux C., Nature, astro-ph/0012222 (2000) 1-20
 - [39] Ostermann P., e-print physics/0211054 (2002) 1-44